SUPPLEMENTARY MATERIAL of "Fast Adaptive Mesh Augmented Lagrangian Digital Image Correlation"

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(Dated: August 12, 2020)

S1. DIC CHALLENGE SAMPLE 14 L5 EXACT DEFORMATION FIELDS

The Society for Experimental Mechanics (SEM) DIC Challenge Sample 14 exact deformation field of synthetic case L5 is sinusoidal with varying frequency in the x-direction, where the exact horizontal x-displacement and infinitesimal strain e_{xx} fields are shown in Fig. S1.

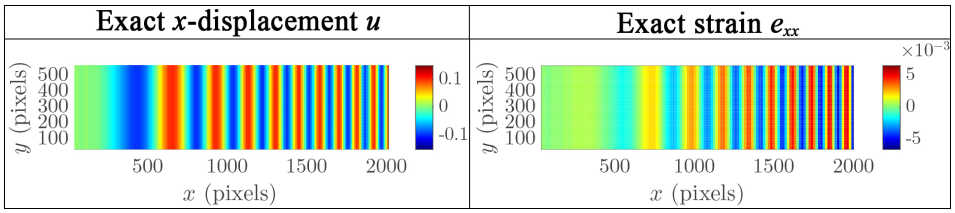

Fig. S1. Exact horizontal x-displacement (left) and strain e_{xx} (right) field associated with Sample 14 L5 (adapted from [1]).

S2. *A POSTERIORI* ERROR ESTIMATES OF SEM 2D-DIC CHALLENGE SYNTHETIC IMAGES OF SAMPLE 14 L5 IN ADAPT-ALDIC AND ADAPTIVE FE-BASED GLOBAL DIC METHODS

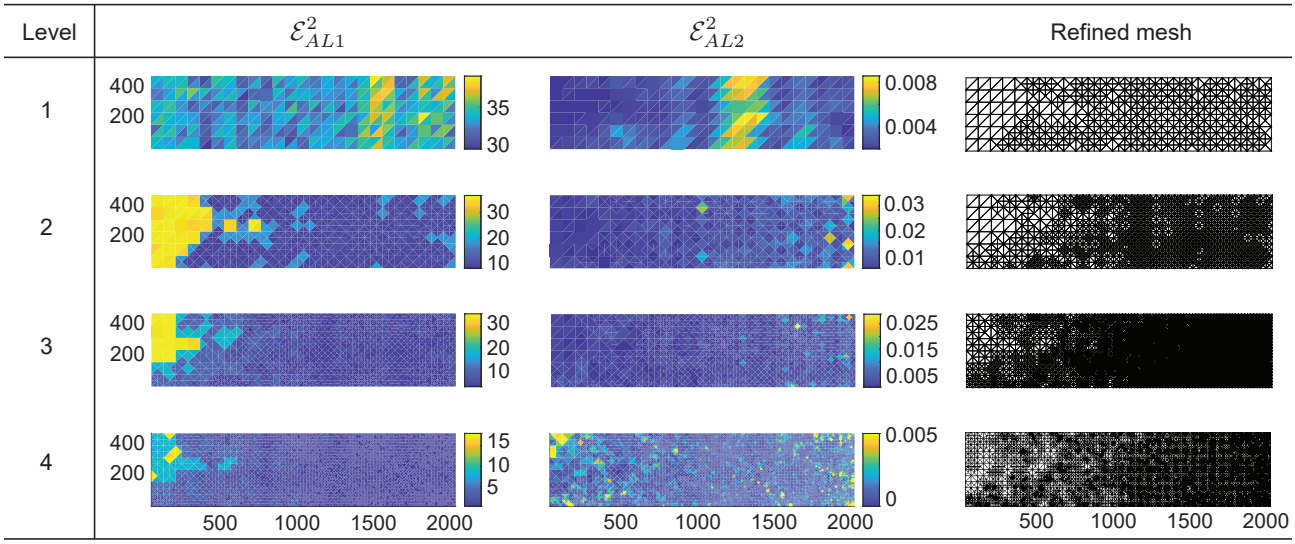

Fig. S2. *A posteriori* error estimates of SEM 2D-DIC Challenge synthetic images of Sample 14 L5 based on Kuhn triangulation adapt-ALDIC subproblem 1 and subproblem 2 and adaptively refined meshes at different mesh levels.

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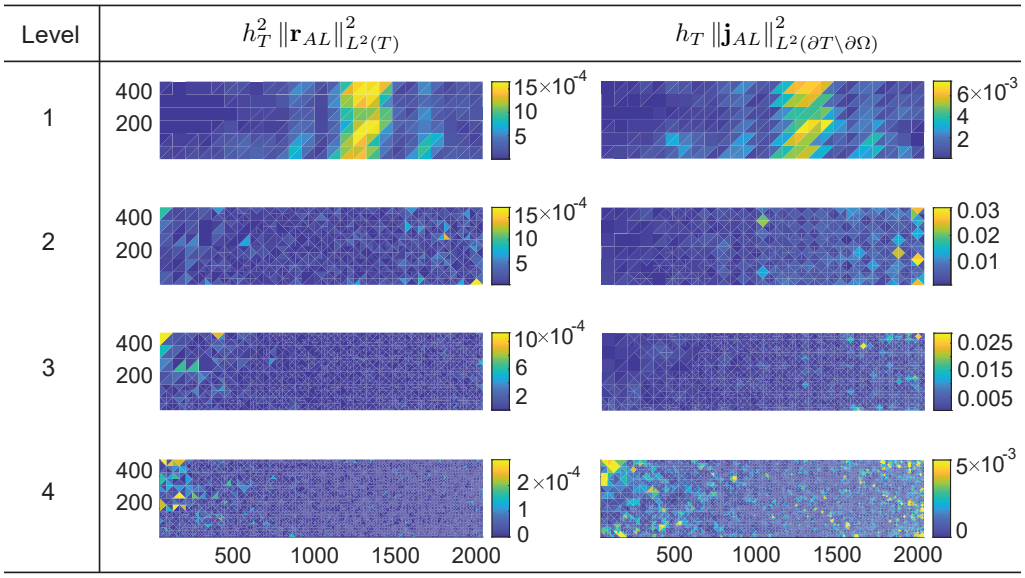

Fig. S3. Interior and jump residuals in the adapt-ALDIC subproblem 2 *a posteriori* error estimates of SEM 2D-DIC Challenge Sample 14 L5 with Kuhn triangulation mesh.

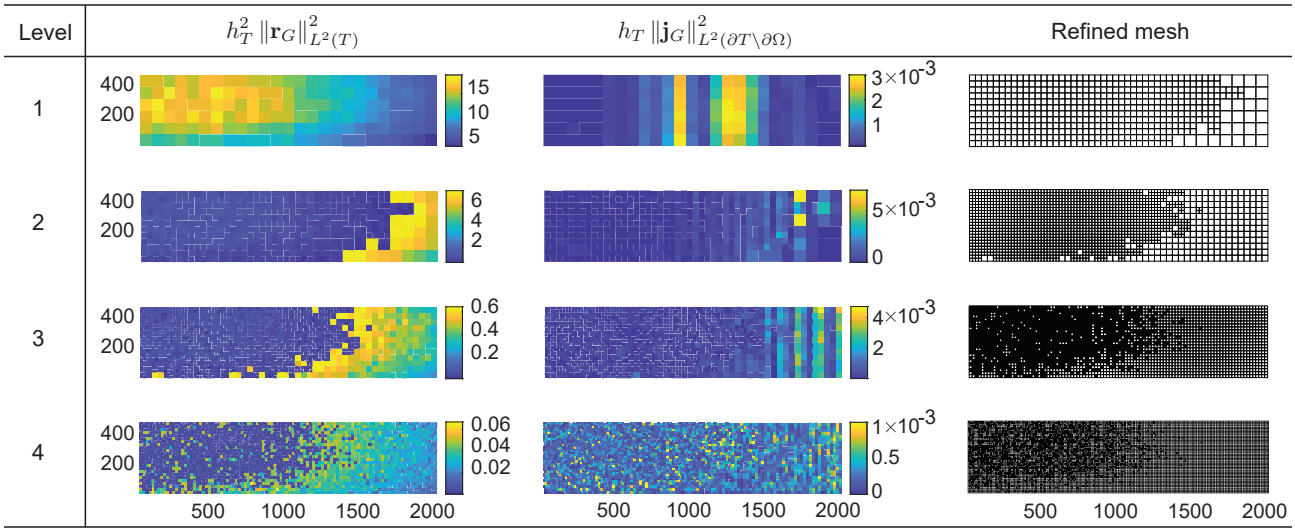

Fig. S4. Interior and jump residuals in the FE-based global DIC method *a posteriori* error estimates, and refined adaptive quadtree meshes of SEM 2D-DIC Challenge Sample 14 L5.

S3. COMPARISON OF NUMBER OF MESH NODES AND ITERATION STEPS IN CASE STUDIES

We show number of mesh nodes (N) and iteration steps in main text Section 4 case studies in Table S1. Symbol k takes the value of k_3, k_4, and 'k_5k_6' in local, FE-based global, and ALDIC methods, respectively.

S4. COMPUTATION TIME COST OF CASE STUDIES USING ADAPT-ALDIC METHOD ON A SINGLE PROCESSOR

We summarize the computation time cost of case studies using adapt-ALDIC method on a single processor in Table S2 .

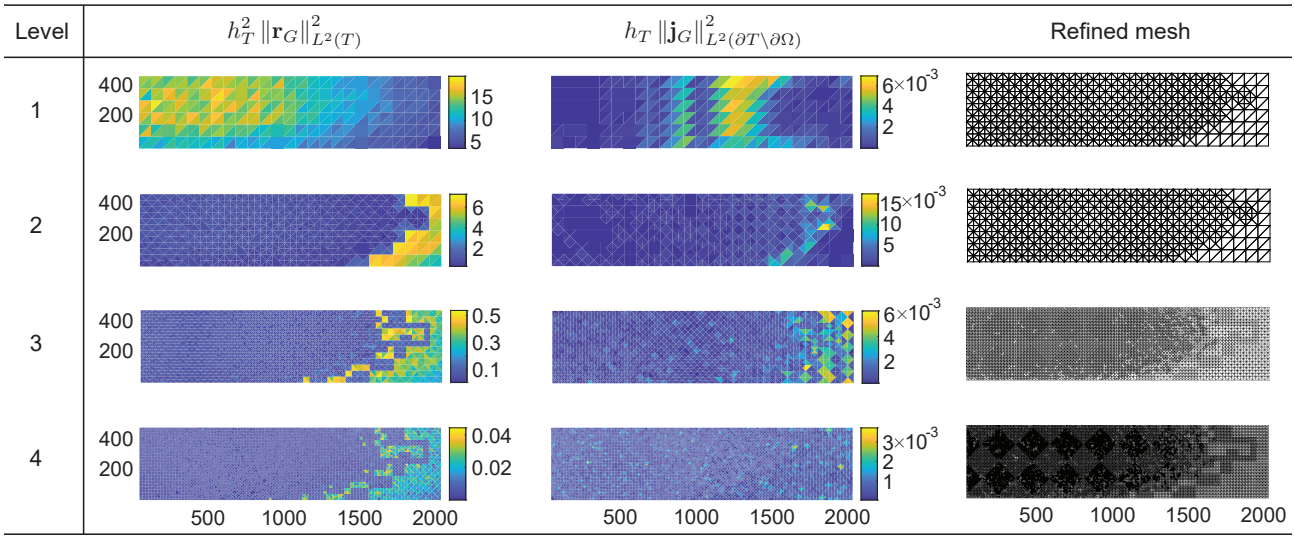

Fig. S5. Interior and jump residuals in the FE-based global DIC method *a posteriori* error estimates, and refined Kuhn triangulation meshes of SEM 2D-DIC Challenge Sample 14 L5.

TABLE S1. Number of mesh nodes (N) and iteration steps in Section 4 case studies

Method name	Mesh adaptivity	Sample 14 L5 (5 px × 5 px) N	Sample 14 L5 (5 px × 5 px) k	cell migration (8 px × 8 px) N	cell migration (8 px × 8 px) k	Shear bands (1 px × 1 px) N	Shear bands (1 px × 1 px) k
Local DIC	Uniform mesh	29889	10.5	62500	20.4	702324	14.4
FE-based global DIC	Uniform mesh	29889	5	62500	21	702324	4
	Adaptive quadtree	19423	3	15691	24	55344	4
	Adaptive Kuhn tri	28337	3	19251	31	12351	6
ALDIC	Uniform mesh	29889	25.3	62500	36.5	702324	20.6
	Adaptive quadtree	13860	25.1	10745	36.0	41814	37.9
	Adaptive Kuhn tri	10900	25.2	8574	114.9	24057	150.3 [a]

TABLE S2. Computation time of uniform and adaptive mesh DIC methods in Section 4 case studies (unit: s)

Method name	Theoretical analysis	Mesh adaptivity	Sample 14 L5 (5 px × 5 px)	Cell migration (8 px × 8 px)	Shear bands (1 px × 1 px)
ALDIC	$O(k_2 k_5 k_6 n_L MNC^{-1})$	Adaptive quadtree	192.64	305.93	3434.8
		Adaptive Kuhn tri	237.29	573.44	2866.4 [a]

[a] Finest triangulation mesh element size could only reach 2 px × 2 px.

[1] PL Reu, E Toussaint, E Jones, HA Bruck, M Iadicola, R Balcaen, DZ Turner, T Siebert, P Lava, and M Simonsen. DIC challenge: Developing images and guidelines for evaluating accuracy and resolution of 2D analyses. *Experimental Mechanics*, 58:1067–1099, 2018.